

Hurst exponent of very long birth time series in XX century Romania. Social and religious aspects

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Abstract

The Hurst exponent of very long birth time series in Romania has been extracted from official daily records, i.e. over 97 years between 1905 and 2001 included. The series result from distinguishing between families located in urban (U) or rural (R) areas, and belonging (Ox) or not (NOx) to the orthodox religion. Four time series combining both criteria, (U,R) and (Ox, NOx), are also examined.

A statistical information is given on these sub-populations measuring their XX-th century state as a snapshot. However, the main goal is to investigate whether the "daily" production of babies is purely noisy or is fluctuating according to some non trivial fractional Brownian motion, - in the four types of populations, characterized by either their habitat or their religious attitude, yet living within the same political regime. One of the goals was also to find whether combined criteria implied a different behavior. Moreover, we wish to observe whether some seasonal periodicity exists.

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The detrended fluctuation analysis technique is used for finding the fractal correlation dimension of such (9) signals. It has been first necessary, due to two periodic tendencies, to define the range regime in which the Hurst exponent is meaningfully defined. It results that the birth of babies in all cases is a very strongly persistent signal. It is found that the signal fractal correlation dimension is weaker (i) for NOx than for Ox, and (ii) or U with respect to R. Moreover, it is observed that the combination of U or R with NOx or OX enhances the UNOx, UOx, and ROx fluctuations, but smoothens the RNOx signal, thereby suggesting a stronger conditioning on religiosity rituals or rules.

1 Introduction

The detrended fluctuation analysis (DFA) method [1] is commonly used to sort out the characteristic Hurst exponent of time series, or its fractal dimension. Alas, many series are of quite finite size. However, the daily birth evolution in Romania during the XX century is known, from official surveys. It amounts to about 35 600 data points, - which an interesting very long time series.

More precisely, the available time series goes from Jan. 01, 1905 till Dec. 21, 2001, i.e. for 97 years, thus 35 429 days; see Sect. 2. Moreover, the birth records distinguish between several social characteristics. In particular, we consider here below the aspects of a urban (U) *vs.* rural (R) family location, and the orthodox (Ox) *vs.* nonorthodox (NOx) family ground. Note that it might be of interest to distinguish cases within the NOx population, i.e. between various religions or sects. Indeed, it has been shown that religious values and practices have some influenced on pregnancy outcomes [2]. In the present case, there are not enough data points for a valuable statistics when considering these sub-populations. Whence we regrouped all non orthodox affiliations into NOx. However, we also study the combination of such "degrees of freedom" location and religiosity. Information on such population distributions in Romania is given in an Appendix (Appendix A) for completeness. We emphasize that the goal is not to report a snapshot of the baby production in Romania in the last century but to examine whether some modelisation is realistic, e.g., through a non-trivial fractional Brownian motion.

In other words, a goal is to investigate whether the "daily" production of babies is purely noisy or is fluctuating, - in four types of populations, characterized by either their habitat or their religious attitude, yet living within the same political regime. For the present paper we focus on considerations, on whether there is a coherent or not behavior in baby births, when either the family location or some family religiosity implies a different collective behavior, and whether the combined set of these two "parameters" (of family characteristics) leads to similar or different outputs. The answer is searched for on whether the various time series correspond to a fractional Brownian Motion [3, 4, 5], characterized by its fractal dimension or its so called Hurst exponent [6].

Another goal was to find whether some periodicity exists, e.g. a seasonal one rather than a yearly one. The DFA is useful in this respect, see Sect. 3.

On the methodology side, recall that the DFA technique [1] is often used to study the correlations in the fluctuations of stochastic time series. It has been much used in econophysics in the recent past [7, 8, 9], but in many other fields as well, as e.g. in meteorology [10], in real signal spectral analysis [11] or faked ones [12]. Briefly speaking, the DFA technique consists in dividing a signal time series $y(t_i)$ of N points, measured at discrete regularly spaced (or not) instants t_i into an integer number of boxes $\simeq N/\tau$ having an equal size ($= n$), such that these non-overlapping boxes (called also windows), each contain τ points [1]. The local trend $z(t_i)$ in each box is assumed to be linear, for simplicity. From the ordinate of a linear least-square fit of the data points in that box, the detrended fluctuation function, $\phi^2(\tau)$, is then calculated following:

$$\phi^2(\tau) = \frac{1}{\tau} \sum_{i=k\tau+1}^{(k+1)\tau} [y(t_i) - z(t_i)]^2 \quad k = 0, 1, 2, \dots, \left(\frac{N}{\tau} - 1\right). \quad (1)$$

Averaging $\phi^2(\tau)$ over the N/τ intervals gives the mean-square fluctuations

$$F(n) = \langle \phi^2(n) \rangle^{1/2} \sim n^\alpha \quad (2)$$

The exponent α , is considered to be identical to the Hurst exponent when the data is stationary; see Sect. 3 for a comment on the presently investigated case. For the reader's own reflexion, it can be here recalled that the (Hurst) exponent of a time dependent signal not only represents the so called *roughness* of the signal but also the behavior of the auto-correlation function $c(\tau)$

$$c(\tau) = \langle |y(t_{i+r}) - y(t_i)| \rangle_\tau, \quad (3)$$

therefore expected to behave like $\sim \tau^{Hu}$ [6].

The output (α) implies the existence of long-range correlations when α is not a half-odd integer. In fact, α is an accurate measure of the most characteristic (maximum) dimension of a fractal process [13]. An exponent $\alpha > 1/2$ implies a *persistent* (smooth) behavior, while $\alpha < 1/2$ means a so-called *anti-persistent* (rough) signal [3]. The latter regime implies a signal fractal dimension D close to 2, the former close to 1. Obviously, the signal fractal (correlation) dimension: $D_2 = 2 - \alpha$, where α is the slope of the $F(n)$ trend, on a $F(n)$ vs. n log-log plot.. The simple Brownian motion is thus characterized by $\alpha = 1/2$ and white noise by $\alpha = 0$ [3]. Practically, the characteristics fractional Brownian motion values are found to lie between 0 and 1 [7, 8, 9]. Our findings are given in Sect. 4.

2 Data

Data was provided by the Romanian National Institute of Statistics (NIS) from the 2002 and 1992 censuses. We have used a query tool available within NIS intranet web site (<http://happy:81/PHC>) regarding the 2002 and 1992 censuses. (A simplified (on a 10% sample) version for that tool is available for

anyone at <http://colectaredate.insse.ro/phc/public.do?siteLang=en>.) The analyzed data represents the number of babies born in Romania and which are still alive at the census in 1992 and in 2002. Data from Jan. 01, 1905 till Dec. 31, 1991 (31776 points) is obtained from the 1992 census; the data from 1992 to 2001 from the 2002 census contains 3653 points.

Two 5 years time interval examples, extracted from such time series, are shown in Fig. 1, on a reasonable scale for better visualization. They correspond to the birth number of babies in *urban* areas between either Jan. 01, 1937 and Jan. 01, 1942, on one hand (red dots) and between Jan. 01, 1965 and Jan. 01, 1970, on the other hand (green dots).

These examples have been selected for showing a state of "nothing too special", although at the WWII time and one "very special" time interval, illustrated by a much relevant event in Romania history, corresponding to Ceausescu decree (#770) forbidding abortion, on Oct. 01, 1966. A remarkable peak is observed in the data in the following year. Note that after that decree the cohort born in 1967 doubled compare to the previous year by almost 1 000 babies.

3 Technical points

Several technical points are in order.

- Regression lines, to obtain α in Eq.(2), were obtained through the maximum likelihood method [14, 15] in order to avoid biases as observed in the least-square regressions in the log-log domain [16]. For a discussion of finite sample effects in sequence analysis, e.g. see [17], but it can be reminded from a technical point of view that Wheaton et al. [18] showed that the least squares 2 free parameter fitting processes can be equivalent to the maximum likelihood method when Poisson statistics apply.

For completeness, note that other methods for fitting to the power-law distributions, as expected here, provide biased estimates for the power-law exponent [19].

- Note that 35 429 is almost a prime number; it can only be decomposed into the product 71 x 499; which would mean to have only two boxes for calculating $F(n)$. Therefore, during the DFA procedure, i.e. before decomposing the data series into equal size boxes (before searching for the residuals and averaging, as described in Eq.(1), several data points are sometimes "not considered" in the process, in order to maintain an equal number of points in each box.

Taking all this into account, the n range which has been always examined goes from ~ 6.91 (~ 1000 days) down to ~ 2.30 , (~ 10 days), - resulting in a series of 100 different box sizes n .

- Note that one should be aware of some possible yearly or seasonal periodicity in the data: it is of common knowledge that babies are not equally born on each day of a year (see Appendix B). Thus some regularity might

be expected, though with amplitude fluctuations over the considered time interval, about one century. On the other hand, it is of common knowledge that every calendar year in Romania starts (in the Gregorian calendar) on Jan. 01, and has "usually" 365 days. There are 24 occurrences of a leap year, but the implications seem marginal.

Every year, there are also 12 months in a year. This is roughly seen on Fig. 1. It has been shown by Hu et al. [20] that a periodicity in a time series implies a break (a change in slope) in the $F(n)$ plot at the n value corresponding to the period. Thus, a "break in slope" at 365 days $\rightarrow n = 5.90 \simeq \ln(365)$ has to be expected in the subsequent plots. Moreover, seasonality might be a sub-period constraint, i.e. 3 months (or $\simeq 90$ days) $\rightarrow n = 4.5 (\simeq \ln(90))$ or even 4 months (or $\simeq 120$ days) $\rightarrow n = 4.8 (\simeq \ln(122))$. To demonstrate the possible periodicity in the present investigation, we have imagined a time series with a constantly increasing amplitude, i.e. adding (i) a constant step each day over 365 or 366 days, depending on the year, like a series of step functions leading to a regular staircase, or (ii) a constant step during each month (with various days) of the 97 years of interest, leading also to a staircase. Fig. 2 shows the periodicity effect in the (i) and (ii) time series on a $F(n)$ vs. n log-log plot. A well marked slope break, from 1.4 to ~ 0 , is obvious at $n \simeq 5.9$ for the (i) time series, $\rightarrow \sim 365$ days. A break in slope is also well observed at $n = 5.9$ for the (ii) time series. This, together with Hu et al. [20] considerations, suggests an upper range analysis limit to be $n \leq 6.0$. The lower range limit is less obvious.

Another *independent* illustration test of a seasonal periodic trend can be performed on some average daily temperature, in order to observe the lower n range limit. For these, our source is the Romanian National Meteorological Agency¹. We chose to test the DFA application to the temperature data series, pertinent to Bucharest Airport. This is illustrated in Fig. 3. Three power law regimes are outlined. Two, the extreme regimes, large n and low n , are giving absurd results. Thereby the lower range limit of interest is found to be at $n = 4.5 (\rightarrow \sim 90$ days, corresponding to a trimester.

Thus, only the DFA data points between $n = 4.5$ and 5.9 should be taken into account for measuring a meaningful Hurst exponent. Therefore, this periodicity elimination criterion has been used for defining a valid time window range in the birth data series analysis.

- Moreover, a question is often raised for statistical purposes, i.e. whether the data is stationary or not [21]. This theoretical question seems somewhat practically irrelevant in many cases, like finance, meteorology, and demography, because the data is "obviously" never stationary. A restricted criterion on stationarity seems likely sufficient for discussion and scientific progress: if the data mean *and* the whatever-extracted-parameter

¹ via [http : //www.ecad.eu/dailydata/customquery.php](http://www.ecad.eu/dailydata/customquery.php)

do not change too much, allowing for a moderate trend, as a function of time, the data can be called quasi-stationary. Nevertheless the matter can be considered thereafter to be the source of a fundamental investigation. The prefix "quasi" is in fact, practically, conventionally dropped. Not having *a posteriori* noticed any specific effect which could be called spurious or anomalous, we have considered our study and findings worth of report, according to the standards mentioned here above.

Therefore, we have made some stationarity test [22, 23, 24]. All series seem to be stationary in this respect. Results of these tests can be provided if needed, but are not displayed in order not to overload the present report.

4 Discussion

The enclosed figures, Figs. 4-5, provide the relevant results of the present study. They report successively the best fits of the relevant $F(n)$ function on a log-log plot and the deduced slope, corresponding to α and equivalently to the Hurst exponent in the following cases, together with the regression coefficient:

- (All) number of babies born from all Romanian families over most of the XX-th century: Figs. 4-5,
- (U) number of babies born from Romanian families in *urban* locations: Fig. 4,
- (R) number of babies born from Romanian families in *rural* locations: Fig. 4,
- (Ox) number of babies born in Orthodox families: Fig. 4,
- (NOx) number of babies born in Non-orthodox (NOx) Romanian families: Fig. 4;
- (UOx) number of babies born from Orthodox families in *urban* locations: Fig. 5,
- (UNOx) number of babies born from Non-Orthodox families in *urban* locations: Fig. 5,
- (ROx) number of babies born from Orthodox families in *rural* locations: Fig. 5,
- (RNOx) number of babies born from Non-Orthodox families in *rural* locations: Fig. 5.

All α values for these different cases are summarized in Table 1. It can be observed that the regression coefficient is quite large, often much larger than 0.96 for the one-criterion filtered data, and above 0.92 for the 2-criteria filtered

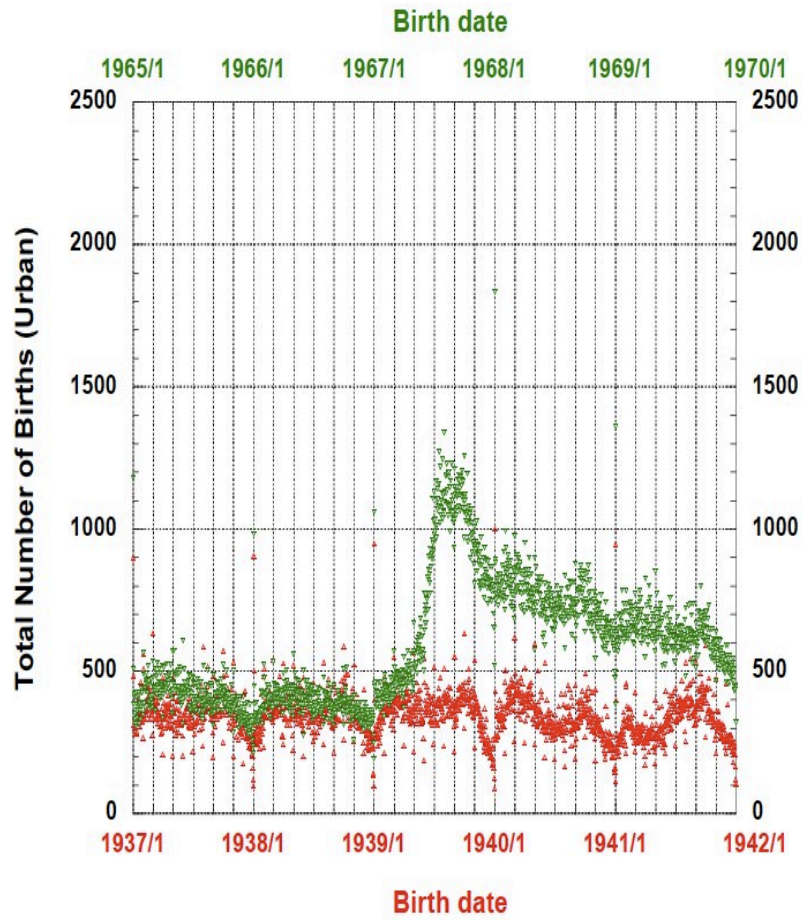


Figure 1: Number of babies born from families in urban locations illustrated as a time series in WWII time [1937-1942] and near Ceausescu forbidding illegal abortion decree (Oct.1, 1966) [1965-1970]

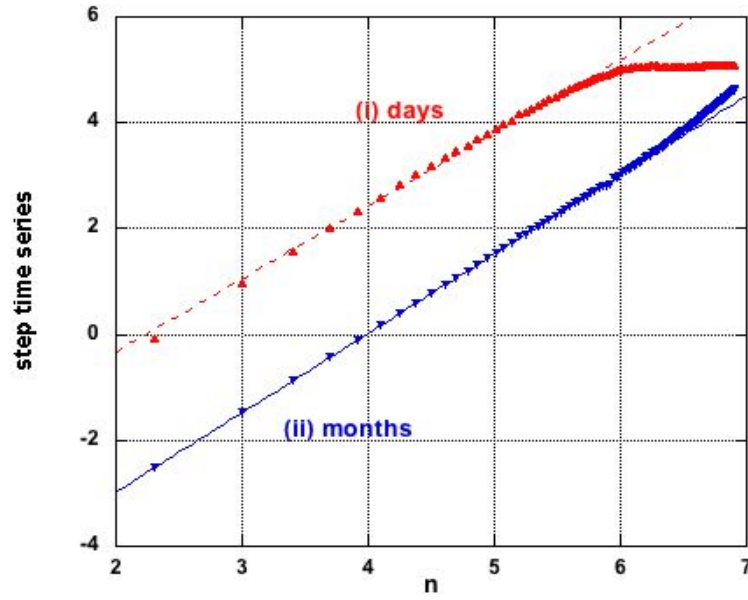


Figure 2: DFA analysis of a staircase function series mimicking the "year" as a function of time, (i) days or (ii) months; a change in slope (from 1.4 to ~ 0) can be observed at $n = 5.9$ for (i), corresponding to ~ 365 days

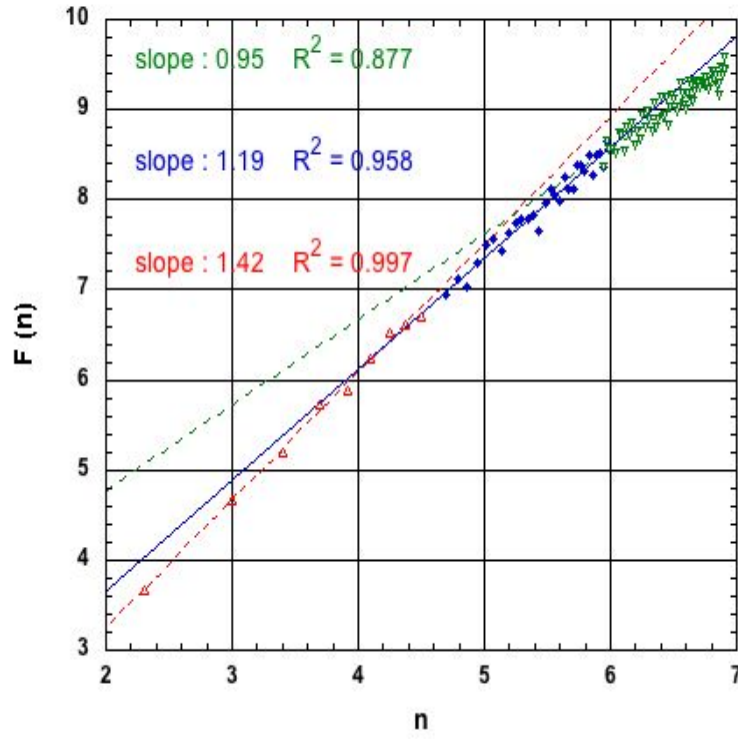


Figure 3: Log-log plot illustrating the DFA result about the yearly periodicity of the daily Temperature in a Romania location (see text) over the 35 429 data points; the 3 fits pertain to different n (box size) regimes: high ($n \geq 6$), medium n and low ($n \leq 4.5$) ranges respectively

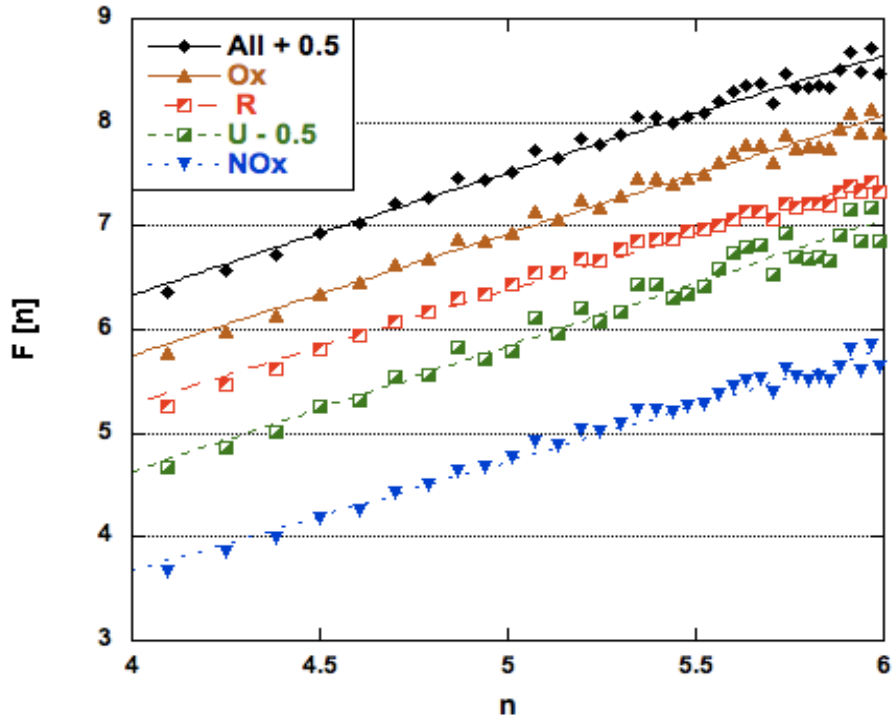


Figure 4: DFA for the daily number of babies born from either Orthodox (Ox) (triangle tip up) or Non-orthodox (NOx) (triangle tip down) Romanian families and of the number of babies born form Romanian families living in either rural (R) (upper left filled square) or urban (U) (lower right filled square) locations in the XX-th century. For readability, the U $F(n)$ amplitude has been displaced; so is the all family case (All) (diamond).

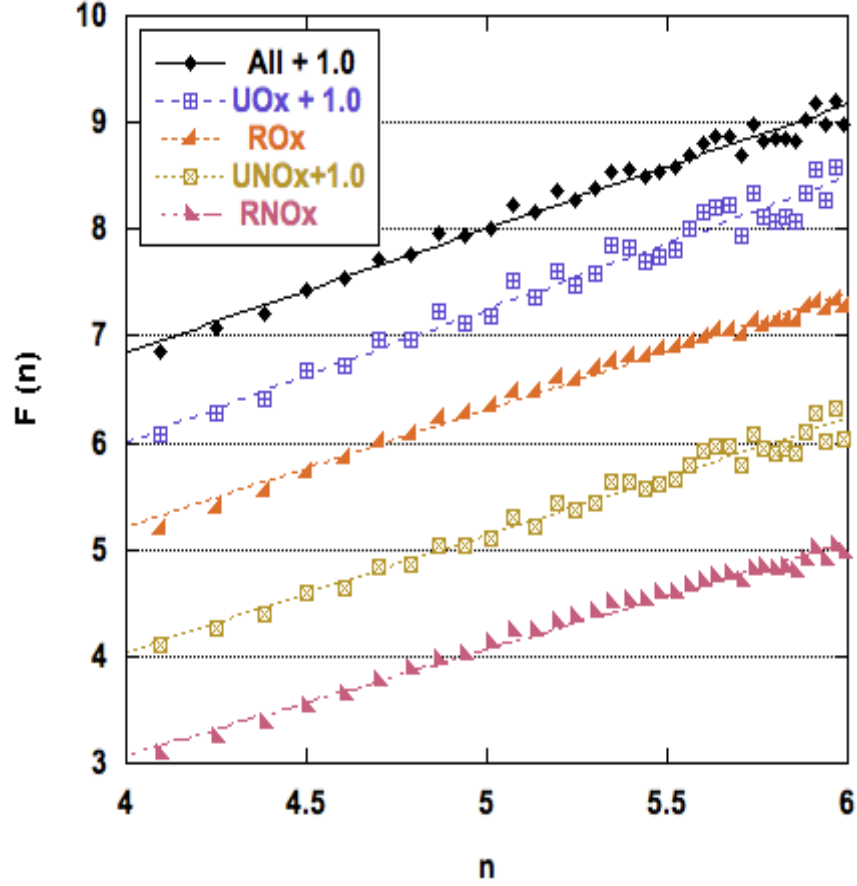


Figure 5: DFA for the daily number of babies born from Orthodox (Ox) or Non-orthodox (NOx) Romanian families living in either rural (R) or urban (U) locations in the XX-th century, i.e. ROx (tip of rectangular triangle to the right), RNOx (tip of rectangular triangle to the left), UOx (+ in square), and UNOx (X-cross in square). For readability, both UOx and UNOx $F(n)$ amplitudes have been displaced; so is the all family case (All) (diamond).

	All	U	R	Ox	NOx.	UOx	UNOx.	ROx	RNOx
D_2	1.132	1.041	1.146	1.166	1.127	1.227	1.206	1.295	1.077
α	0.821	0.959	0.854	0.834	0.873	0.773	0.794	0.705	0.923
$\pm\delta\alpha$	0.021	0.034	0.016	0.022	0.053	0.046	0.056	0.033	0.057
R^2	0.980	0.964	0.989	0.980	0.934	0.927	0.916	0.957	0.934
	Figs. 4-5		Fig. 4			Fig. 5			

Table 1: Resulting values of the fits to the indicated data with a power law, Eq.(2), for obtaining the Hurst exponent α and its error $\delta\alpha$, from which the signal fractal dimension, $D_2 = 2 - \alpha$, can be obtained.

data.. The Hurst exponent usually has either a high value (~ 0.9) close to that of a strict *persistent* signal [3], or is near 1.2, - mostly for the 2-criteria filtered signals. For comparing with a visual case, let it be known that the roughness of the line of summits in the French Alps near Chamonix, FR, has a fractal dimension = 1.2.

It can be observed that some very slight difference exists between Ox and NOx; it might be partially due to the amplitude values, as well as between rural or urban locations, for the same reason. In such cases, the signal fractal dimension is always weaker for NOx than Ox, and for R with respect to U.

The most interesting point is found when observing the fractal dimension or Hurst exponent when two criteria are imposed: the birth in UOx (urban & orthodox) families becomes less coherent ($D_2 \geq 1.2$), while RNOx (rural & non-orthodox) families) tends toward a smoother persistent behavior.

5 Conclusions

This report has presented a study of the behaviors of populations according to long time series data. One of the goals was to investigate whether the "daily" production of babies is a purely noisy signal or is fluctuating according to some non trivial fractional Brownian motion. We investigated various types of populations, characterized by either their habitat or their religious attitude, yet living within the same political regime. We also searched whether some periodicity exists in such signals.

We describe the data and the methodology. We could analyze very long time series giving the number of babies born per day in Romania during the XX century. We used a simple DFA method on all together 9 cases. In conclusion, it has been found that beyond the signal periodicity and the various historical and political events, the correlations between births lead to a very persistent signal, - whatever the family location or religious membership, even though the data looks very stochastic.

We found collective coherence. However, we have also shown a combined criterion effect. The Non-orthodox sub-population living in urban areas, has a

behavior similar to that of the Orthodox population, wherever it is located, but the RNOx population departs from a usual random process, - for us implying a stronger conditioning on religiosity rituals or rules.

For further studies, we may suggest (as also recommended by a reviewer) to go beyond the simple DFA, in order to investigate what power-law cross correlations can be found between such simultaneously recorded time series, thus generalizing Eq.(3). To do so, the Detrended Cross-Correlation Analysis (DCCA) [25, 26, 27, 28], also including multifractality aspects [29, 30, 31, 32], can be useful. However, this leads to different aspects than those hereby searched for and outlined.

Acknowledgment

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Appendix A: Statistical summary on population communities

A few general information details about population ethnicity, structure by urban/rural, gender, religion, in Romania could serve to enlighten considerations by the reader.

The most important community, from the religion point of view, according to the census sources is the Eastern-Orthodox (86.8%). The so called "non-orthodox families" (13.2%) are mainly made of Roman-Catholics (4.7%), Reformations (3.2%), Pentecostals (1.5%), Greek-Catholics (0.9%), Baptists (0.6%), Seventh Day Adventists (0.4%), Moslems (0.3%), Unitarians (0.3 %), Lutherans (0.3%), Evangelicals (0.2%) and Old rite Christians (0.2%). Other denominations, including atheists, have each smaller a size.

From the geographical point of view the ratio between rural and urban population was $\simeq 0.8708$ *ca.* 2000. It has evolved with time, of course. Indeed, the urbanity dramatically increased in the 5-6th decade of the last century with forced industrialization imposed by communists. Many villages were suddenly declared as towns.

Moreover, from the ethnic point of view the most important communities, according to the 2002 Census official data, are: Romanians (89.5%), Hungarians(6.6%), Rromans (\equiv Gypsies) (2.5%), Ukrainians(0.3%), Germans (0.3%), Russians (0.2%), Turks (0.2%), Tatars(0.1%) and Serbians (0.1%).

The intersection of such sets can be described in a statistical way through distribution characteristics; see Table 2. Only a few selected groups are mentioned. Notations seem obvious. More information can be obtained from the authors if necessary, upon request.

Appendix B: Seasonal effects on births

Babies are not regularly born during the years. The phenomenon has always existed, but the peaks have evolved with time, due to socio-economic, political

	min	Max	Sum	mean	med.	Std. Dev.	Skewn.	Kurt.
all U	5	2158	13335045	376.388	358	206.39	0.371	0.880
all U rmn	3	1910	11604268	327.536	306	185.44	0.405	0.737
all U hung	0	78	360793	10.184	3	12.95	1.308	0.811
all U gyps	0	35	197990	5.588	4	5.49	0.965	0.268
all U germ	0	11	32560	0.919	0	1.31	1.805	3.743
all R	14	1383	11612016	327.755	336	130.94	0.022	0.835
all R rmn	9	1228	10018082	282.765	289	114.28	0.076	0.754
all R hung	0	85	328882	9.283	2	11.20	0.885	-0.358
all R gyps	0	49	296861	8.379	5	8.14	1.018	0.164
all R germ	0	9	15818	0.447	0	0.83	2.277	6.150
all Ox U	3	1944	11578538	326.810	307	186.96	0.411	0.854
Ox U rmn	3	1992	11672383	329.458	314	189.35	0.433	1.043
Ox U hung	0	145	625732	17.663	18	18.28	0.490	-0.718
Ox U gyps	0	31	191381	5.402	4	4.90	0.784	-0.0087
Ox U germ	0	16	55800	1.575	1	1.97	1.284	1.245
all Ox R	8	1267	10051258	283.701	289	116.22	0.089	0.862
Ox R rmn	8	1274	10019369	282.801	282	117.66	0.204	1.069
Ox R hung	0	113	457184	12.904	12	13.54	0.515	-0.613
Ox R gyps	0	42	280903	7.929	6	6.83	0.765	-0.166
Ox R germ	0	9	25832	0.729	0	1.16	1.865	3.786

Table 2: A few statistical characteristics of the daily number of births in Romania during the XX-th century according to different criteria for characterizing families, not only location and religiosity, but also ethnicity

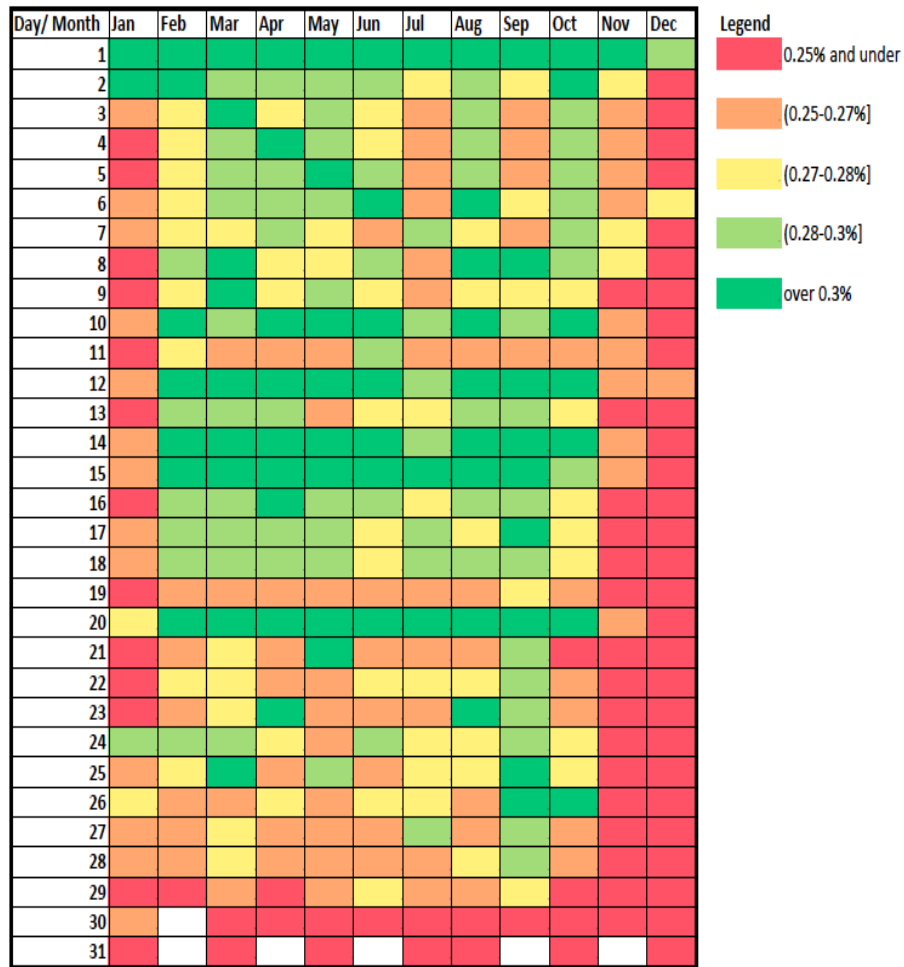


Figure 6: Table of the daily percentage level of births for the 97 years studied in the main text, showing seasonal effects.

and religiosity conditions. It is known that the phenomenon mainly depended on seasons, because of night duration and temperature. Wargentin [33] showed that the baby production was more frequent in december than in other months; he suggested "causes" for his finding. Quetelet [34] found a birth peak in February and a minimum in July for Belgium and The Netherlands, in [1815-1826]. Moheau [35] already distinguished country side from cities, but found in both cases, a birth peak in spring time, thus a summer production. Villermé [36] pretended that the cause for such variations was the temperature. Nowadays, i.e. in present times, the temperature cause has to be much disregarded [37].

The seasonality effect is less marked nowadays in advanced civilizations. Nevertheless, it is still observed, e.g. in Romania, as shown on Fig. 6. The figure results from a daily aggregation for all 97 years, whence the data concerns almost $25 \cdot 10^6$ births. After aggregating the number of births data for each day, the daily shares were calculated. Next the values were coded within their quintiles; different colors are used to distinguish different groups. It can be understood that a depletion during winter months (December-February) implies a lack of production at the end of winter and beginning of spring, when work activity in the fields has to resume. Recall that a uniform distribution requires for each day a $1/365.25$ share, i.e. $\simeq 0.2732785\%$. N.B. We have applied a χ^2 test for comparing to a uniform distribution and found statistically significant differences: $p \leq 0.001$.

Notice that we have investigated, as a possible "parameter", some religious affiliation which implies some specific sexual behavior at various times during the year, thus leading to something else than a seasonal effect. It is known that religious rituals are quite tied to seasonal activities. Therefore, a weaker attachment to religious principles might modify the seasonality effect [37]. However, a Fourier transform of the examined data in the main text and an extraction of whatever cycles are not part of the present investigations, only concerned by whether the daily birth production is a coherent or not signal.

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